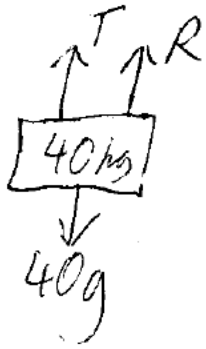
M1 Exam June 2004

(1)

Block at rest so $T = 10g$ N.



Block at rest,

so $40g = T + R$

$= 10g + R$

so $\boxed{R = 30g = 294\text{ N}}$

(2) (i) Steepest part of graph shows fastest speed, & is straight when $4 \le t \le 6$.

Speed $= \dfrac{7-3}{2} = 2\text{ ms}^{-1}$

(ii) 4 seconds

(iii) Slope on "return" is $\dfrac{10}{20-14} = \dfrac{10}{6} = 1\tfrac{2}{3}\text{ ms}^{-1}$

(3) $v = \dfrac{dx}{dt} = 0.12t - 0.0003t^2$

Train is at rest at A & B, so $v=0$ at these points. So we need t when $v=0$:-

$0 = 0.12t - 0.0003t^2$

$\Leftrightarrow \quad 0 = t(12 - 0.03t) \Leftrightarrow$ $t = 0$ or $t = \dfrac{12}{0.03} = 400$

PTO

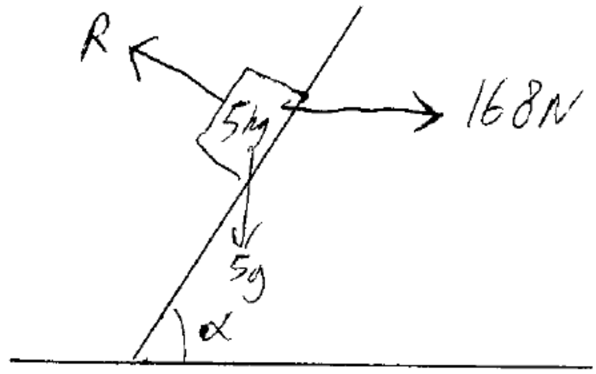
(3) (cont) So train is at rest at $t=0$ & $t=$

so takes $\boxed{400s}$ to get from A to B.

(ii) When $t = 400$

$$x = 0.06 \times 400^2 - 0.0001 \times 400^3$$
$$= \boxed{3200m}$$

(4) (i)



• Component of weight in direction of plane:-

$5g \sin\alpha$

• Component of horizontal force in direction of plane:-

$168 \cos\alpha$.

As block is at rest these forces cancel each other, & so

$$168 \cos\alpha = 5g \sin\alpha$$

ie. $\frac{\sin\alpha}{\cos\alpha} = \tan\alpha = \frac{168}{5g} = 3.429$ (to 3dp)

So $\alpha = \tan^{-1}(3.429) = \boxed{73.7^\circ}$

(4) (ii) The reaction from the plane must c... the components of the weight & the horizon... force perpendicular to the plane.

So, $R = 5g\cos\alpha + 168\sin\alpha$

$= \boxed{175N}$

(iii) Now use Newtons 2^{nd} Law $(F=ma)$ to get:-

$5g\sin\alpha = 5a$

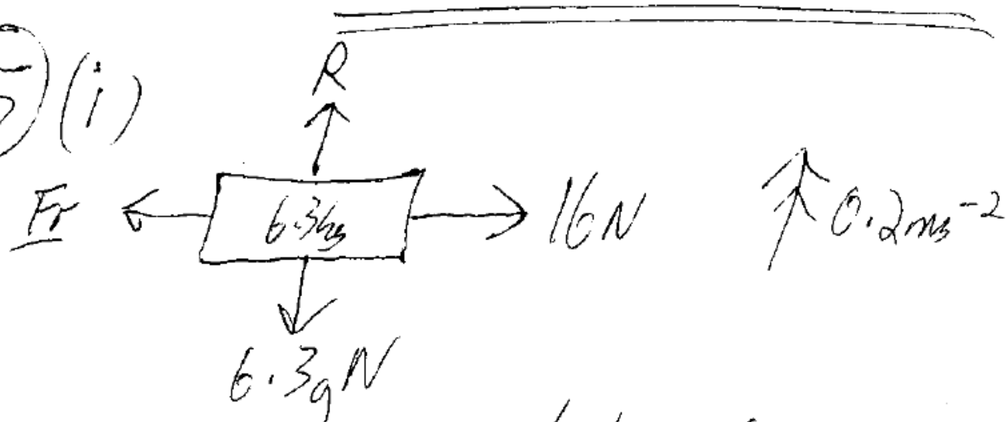
So $a = \dfrac{5\times 9.8\times \sin(73.7)}{5} = \underline{9.41 ms^{-2}}$ (to 3 s.f.)

Finally use $s = ut + \frac{1}{2}at^2$ to give:-

$s = 0 + \frac{1}{2}\times 9.41\times 0.8^2$

$= \boxed{3.01m}$ (to 3 s.f.)

(5) (i)



Using Newtons 2^{nd} law $(F=ma)$ gives:-

$R - 6.3g = 6.3\times 0.2$

So $R = \underline{63N}$

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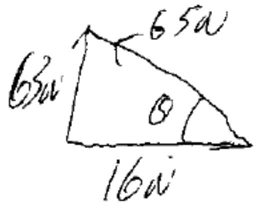
5) (cont) As there is no horizontal movement $Fr = 16N$.

Thus, using Pythagoras, the resultant of the reaction & friction is:-

$$\sqrt{16^2 + 63^2} = \boxed{65N}$$



$$\theta = \tan^{-1}\left(\frac{63}{16}\right) = \boxed{75.7^\circ}$$

(to 3s.f)

(ii) On the point of slipping $Fr = \mu R$, & $X = Fr$, so

$$X = 0.3 \times 63$$

$$= \boxed{18.9N}$$

(iii) As the lift is moving downward the reaction between the floor & the box is reduced, & so friction is reduced. Thus the horizontal force needed to move the box is less than X N.

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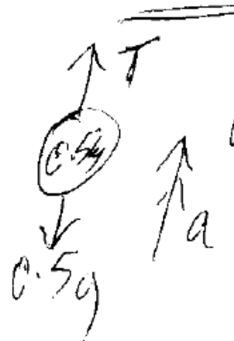
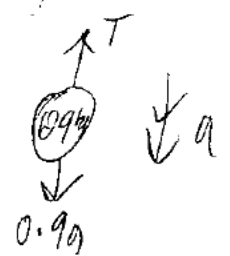
6) B :-



Using $F=ma$:- $T - 0.5g = 0.5a$ (1)

Again using $F=ma$:-

A :-



$0.9g - T = 0.9a$ (2)

PTO

⑥ (cont) Adding equations (1) & (2) gives:-

$$0.4g = 1.4a$$

So $\boxed{a = 2.8\,ms^{-2}}$

Thus (1) gives:- $T = 0.5g + 0.5 \times 2.8$
$= \boxed{6.3N}$

(ii) Using $s = ut + \frac{1}{2}at^2$ we get that A moves
$s = 0 + \frac{1}{2} \times 2.8 \times 0.8^2 = \underline{0.896m}$
downwards, before hitting the block.

B continues to move, initially at speed given by $v = u + at$
$= 0 + 2.8 \times 0.8 = \underline{2.24\,ms^{-1}}$,
but with negative acceleration of $9.8\,ms^{-2}$.

So when B instantaneously stops it will have travelled a further distance given by:-

$$v^2 = u^2 + 2as$$

$$0 = 2.24^2 + 2 \times (-9.8) \times s$$

So $s = \frac{2.24^2}{19.6} = \underline{0.256m}$

So total height of B is $0.256 + 0.896$
$= \boxed{1.152m}$

(iii) Using $s = \frac{1}{2}(u+v)t$ we can see A is on the block for $t = \frac{2s}{u+v} = \frac{2 \times 0.256}{2.24} = 0.229s$
while B is continuing upward.

(PTO)

6 (cont) But it will take the same time for B to descend the same distance, so the total time A is on the block is $2 \times 0.229 = \boxed{0.457\,s}$

7 (i) Conservation of momentum gives:-

$$(8 \times 3.6) + (0 \times 3.2) = (2.4 \times 3.6) + (v \times 3.2)$$

So $\boxed{v = 6.3\,ms^{-1}}$

(ii) $s = 17m$, $u = 6.3\,ms^{-1}$ $v = 5.6\,ms^{-1}$ $a = ?$

$\underline{v^2 = u^2 + 2as}$ gives:- $5.6^2 = 6.3^2 + 2 \times 17 \times a$

So $a = \underline{-0.245\,ms^{-2}}$

Now $F = ma$ gives $F = 0.32 \times 0.245$
$= \underline{0.0784\,N}$

& $F_r = \mu R$ gives $0.0784 = \mu \times (0.32g)$
(since $F = F_r$ & $R =$ weight)

So $\boxed{\mu = 0.025}$

(iii) $s = 4m$, $a = -0.245$, $v = 0$, so

$u = \sqrt{(2 \times 4 \times 0.245)} = \underline{1.4\,ms^{-1}}$ (from $v^2 = u^2 + 2as$)

Before collision B has momentum:-
$5.6 \times 0.32 = 1.792\,Ns$

After collision B has momentum
$-1.4 \times 0.32 = -0.448\,Ns$

So change is $1.792 - (-0.448) = \boxed{2.24\,Ns}$